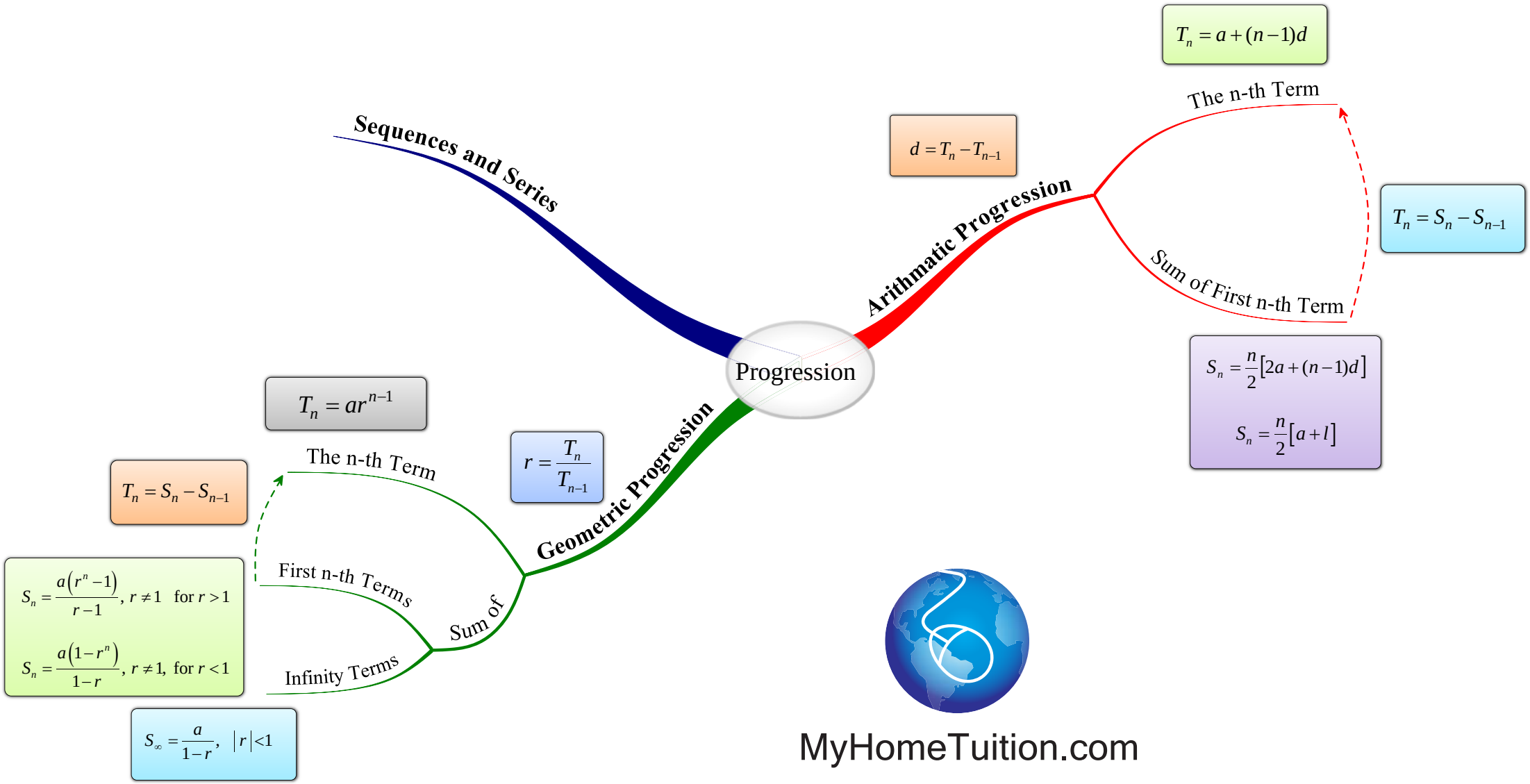

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01 Progressions

Arithmetic Progression	Geometric Progression
1. Common Difference $\boxed{d = T_n - T_{n-1}}$	**1. Common Ratio** $\boxed{r = \frac{T_n}{T_{n-1}}}$
2. The *n*th term $\boxed{T_n = a + (n-1)d}$	**2. The *n*th term** $\boxed{T_n = ar^{n-1}}$
3. Sum of the first *n* terms $\boxed{S_n = \frac{n}{2}[2a + (n-1)d]}$ or $\boxed{S_n = \frac{n}{2}[a + l]}$ where l = last term	**3. Sum of the first *n* terms** $\boxed{S_n = \frac{a(r^n - 1)}{r - 1},\ r \neq 1 \quad \text{for } r > 1}$ $\boxed{S_n = \frac{a(1 - r^n)}{1 - r},\ r \neq 1,\ \text{for } r < 1}$ **4. Sum to infinity:** $\boxed{S_\infty = \frac{a}{1 - r},\quad \lvert r \rvert < 1}$

If S_n is given as a function of *n*, then use**:**

$$\boxed{T_n = S_n - S_{n-1}}$$

First term, $\boxed{a = T_1 = S_1}$

Second term, $\boxed{T_2 = S_2 - S_1}$

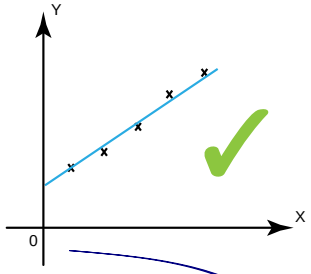
Sum of the terms from *m*th term to *n*th term:)

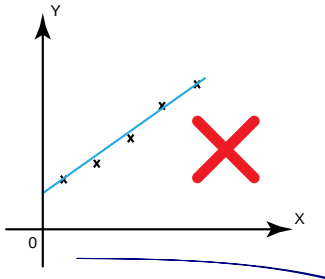
$$\boxed{\text{Sum} = S_n - S_{m-1}}$$

(For example –The sum from 3rd term to 7th term, $Sum = S_7 - S_2$)

Linear Law

Non-linear equation	Linear equation	Y	X	m	c
$y = px^2 + qx$	$\frac{y}{x} = px + q$	$\frac{y}{x}$	x	p	q
	$\frac{y}{x^2} = \frac{q}{x} + p$	$\frac{y}{x^2}$	$\frac{1}{x}$	q	p
$y = px + \frac{q}{x}$	$xy = px^2 + q$	xy	x^2	p	q
	$\frac{y}{x} = \frac{q}{x^2} + p$	$\frac{y}{x}$	$\frac{1}{x^2}$	q	p
$y = p\sqrt{x} + \frac{q}{\sqrt{x}}$	$y\sqrt{x} = px + q$	$y\sqrt{x}$	x	p	q
	$\frac{y}{\sqrt{x}} = \frac{q}{x} + p$	$\frac{y}{\sqrt{x}}$	$\frac{1}{x}$	q	p
$\frac{p}{y} = \frac{q}{x} + 1$	$\frac{1}{y} = \frac{q}{px} + \frac{1}{p}$	$\frac{1}{y}$	$\frac{1}{x}$	$\frac{q}{p}$	$\frac{1}{p}$
$y = \frac{x}{p + qx}$	$\frac{1}{y} = p\left(\frac{1}{x}\right) + q$	$\frac{1}{y}$	$\frac{1}{x}$	p	q
$y = \frac{9}{p^2}(x+q)^2$	$\sqrt{y} = \left(\frac{3}{p}\right)x + \frac{3q}{p}$	$\sqrt{y}$	x	$\frac{3}{p}$	$\frac{3q}{p}$
$y = pq^x$	$\log_{10} y = x\log_{10} q + \log_{10} p$	$\log_{10} y$	x	$\log_{10} q$	$\log_{10} p$
$y = px^q$	$\log_{10} y = q\log_{10} x + \log_{10} p$	$\log_{10} y$	$\log_{10} x$	q	$\log_{10} p$
$y = pq^{x+1}$	$\log_{10} y = (x+1)\log_{10} q + \log_{10} p$	$\log_{10} y$	$x+1$	$\log_{10} q$	$\log_{10} p$





$Y = mX + c$

$Y - y_1 = m(X - x_1)$

Formation of the Equation

Determine the Values of Variables

Line of Best Fit

Linear Law

Applications to Non-Linear Function

Reduce Non-Linear Function to linear Function

Solving Linear Law Problems



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02 Linear Law

1. *Drawing lines of best fit*

Line of best fit has 2 characteristics:

1. it passes through as many points as possible,
2. the number of points which are not on the line of best fit are equally distributed on the both sides of the line.

Steps to draw a line of best fit:

1. Construct a table consisting the given variables.
2. Plot a graph of Y against X , using the scale specified AND draw a line of best fit.
3. Calculate the gradient, m, and get the Y-intercept, c, from the graph.
4. Re-write the original equation given and reduce it to linear form.
5. Compare with values of m and c obtained, find the values of the unknowns required.

2. *Equations of lines of best fit*

1. A set of two variables are related non linearly can be converted to a linear equation. The line of best fit can be written in the form

$$\boxed{Y = mX + c}$$

where
X and Y are in terms of x and/or y
m is the gradient,
c is the Y-intercept

2. The graph of $Y = mX + c$ can be used to find the values of constants of the non-linear equation and others information relating the two variables.

Recall:

(1) Equation of a straight line if two points are given:

$$\boxed{Y - y_1 = m(X - x_1)}$$

(2) Equation of a straight line if m and c are given:

$$\boxed{Y = mX + c}$$

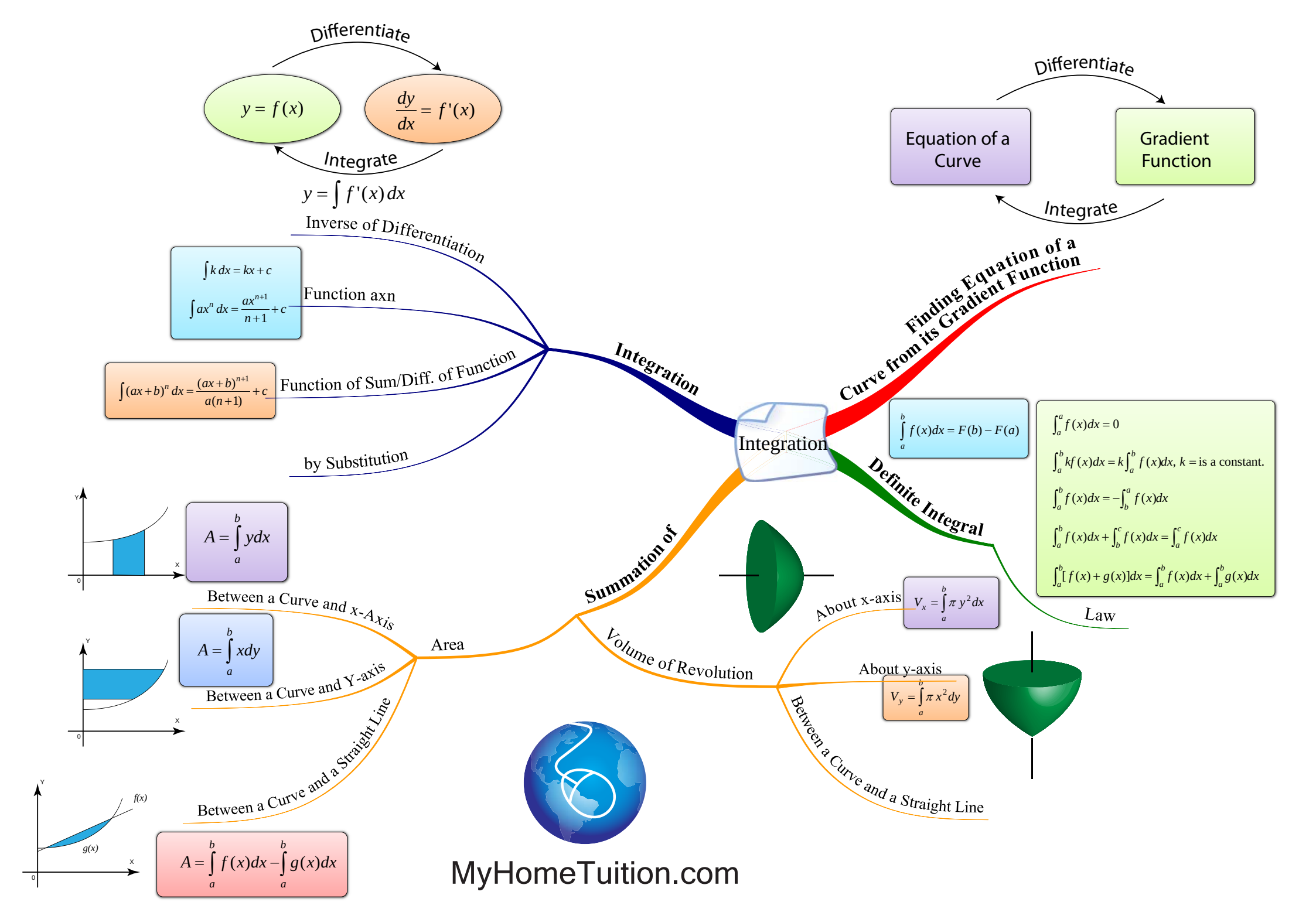
3. *To reduce non-linear functions to linear form*

Tips:

(1) The equation must have one constant (without x and y).
(2) Y cannot have constant, but can have x and y.
(3) X cannot have y., but can have x and constant.

The table shows some of examples of non-linear equations can be reduced to the linear form:

Non-linear equation	**Linear equation**	***Y***	***X***	***m***	***c***
$y = px^2 + qx$	$\frac{y}{x} = px + q$	$\frac{y}{x}$	x	p	q
	$\frac{y}{x^2} = \frac{q}{x} + p$	$\frac{y}{x^2}$	$\frac{1}{x}$	q	p
$y = px + \frac{q}{x}$	$xy = px^2 + q$	xy	x^2	p	q
	$\frac{y}{x} = \frac{q}{x^2} + p$	$\frac{y}{x}$	$\frac{1}{x^2}$	q	p
$y = p\sqrt{x} + \frac{q}{\sqrt{x}}$	$y\sqrt{x} = px + q$	$y\sqrt{x}$	x	p	q
	$\frac{y}{\sqrt{x}} = \frac{q}{x} + p$	$\frac{y}{\sqrt{x}}$	$\frac{1}{x}$	q	p
$\frac{p}{y} = \frac{q}{x} + 1$	$\frac{1}{y} = \frac{q}{px} + \frac{1}{p}$	$\frac{1}{y}$	$\frac{1}{x}$	$\frac{q}{p}$	$\frac{1}{p}$
$y = \frac{x}{p + qx}$	$\frac{1}{y} = p\left(\frac{1}{x}\right) + q$	$\frac{1}{y}$	$\frac{1}{x}$	p	q
$y = \frac{9}{p^2}(x + q)^2$	$\sqrt{y} = \left(\frac{3}{p}\right)x + \frac{3q}{p}$	$\sqrt{y}$	x	$\frac{3}{p}$	$\frac{3q}{p}$
$y = pq^x$	$\log_{10} y = x \log_{10} q + \log_{10} p$	$\log_{10} y$	x	$\log_{10} q$	$\log_{10} p$
$y = px^q$	$\log_{10} y = q \log_{10} x + \log_{10} p$	$\log_{10} y$	$\log_{10} x$	q	$\log_{10} p$
$y = pq^{x+1}$	$\log_{10} y = (x+1) \log_{10} q + \log_{10} p$	$\log_{10} y$	$x + 1$	$\log_{10} q$	$\log_{10} p$


Differentiate
$y = f(x)$
$\frac{dy}{dx} = f'(x)$
Integrate
$y = \int f'(x)\,dx$
Inverse of Differentiation
$\int k\,dx = kx + c$
$\int ax^n\,dx = \frac{ax^{n+1}}{n+1} + c$
Function axn
$\int (ax+b)^n\,dx = \frac{(ax+b)^{n+1}}{a(n+1)} + c$
Function of Sum/Diff. of Function
by Substitution
Integration
Differentiate
Equation of a Curve
Gradient Function
Integrate
Finding Equation of a Curve from its Gradient Function
$\int_a^b f(x)dx = F(b) - F(a)$
Definite Integral
$\int_a^a f(x)dx = 0$
$\int_a^b kf(x)dx = k\int_a^b f(x)dx$, k = is a constant.
$\int_a^b f(x)dx = -\int_b^a f(x)dx$
$\int_a^b f(x)dx + \int_b^c f(x)dx = \int_a^c f(x)dx$
$\int_a^b [f(x) + g(x)]dx = \int_a^b f(x)dx + \int_a^b g(x)dx$
Law
Summation of
Area
Between a Curve and x-Axis
$A = \int_a^b y\,dx$
Between a Curve and Y-axis
$A = \int_a^b x\,dy$
Between a Curve and a Straight Line
$A = \int_a^b f(x)dx - \int_a^b g(x)dx$
Volume of Revolution
About x-axis
$V_x = \int_a^b \pi y^2 dx$
About y-axis
$V_y = \int_a^b \pi x^2 dy$
Between a Curve and a Straight Line
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03 Integration

1. Integration is the inverse process of differentiation.
   If $\boxed{\frac{dy}{dx} = f'(x)}$, then $\boxed{\int f'(x)dx = y + c}$ where c = constant.

2. Indefinite integrals:(Refer to the examples below after note 6)
   (a) $\boxed{\int k\,dx = kx + c}$

   (b) $\boxed{\int ax^n\,dx = \frac{ax^{n+1}}{n+1} + c}$

   (c) $\boxed{\int (ax+b)^n\,dx = \frac{(ax+b)^{n+1}}{a(n+1)} + c}$

3. Definite integrals
   $\boxed{\int_a^b f(x)dx = F(b) - F(a)}$

   The laws of definite integrals:

   (a) $\int_a^a f(x)dx = 0$

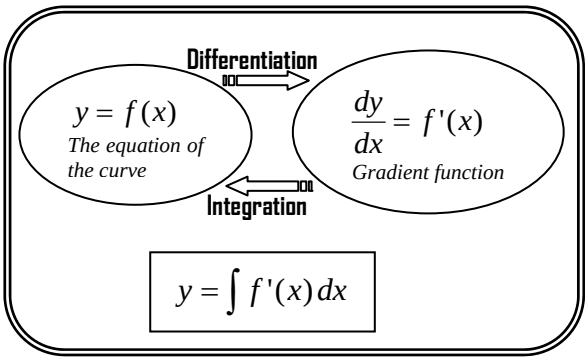
   (b) $\int_a^b kf(x)dx = k\int_a^b f(x)dx$, $k =$ is a constant.

   (c) $\int_a^b f(x)dx = -\int_b^a f(x)dx$

   (d) $\int_a^b f(x)dx + \int_b^c f(x)dx = \int_a^c f(x)dx$

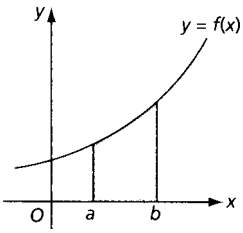
   (e) $\int_a^b [f(x) + g(x)]dx = \int_a^b f(x)dx + \int_a^b g(x)dx$

4. Finding Equation Of A Curve From Its Gradient Function:



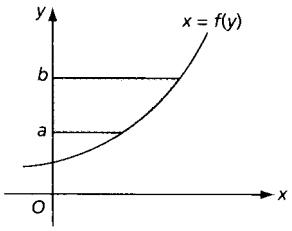
5. Integration As The Summation Of Areas:

Area bounded by the curve y= f(x),the lines x = a, x = b and the x-axis



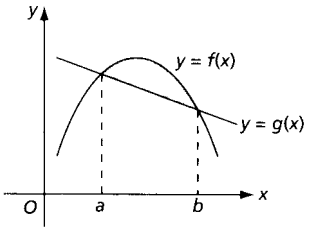
$$A = \int_a^b y\,dx$$

Area bounded by the curve x = f(y), the lines y = a, y = b and the y-axis.



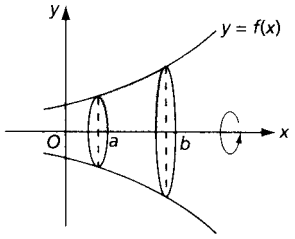
$$A = \int_a^b x\,dy$$

Area of the region between a curve y = f(x) and a straight line y = g(x)



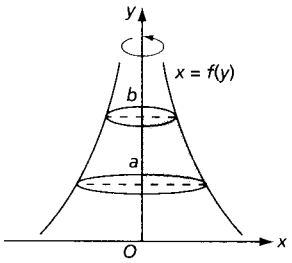
$$A = \int_a^b f(x)\,dx - \int_a^b g(x)\,dx$$

6. Integration As The Summation Of Volumes



The volume of the solid generated when the region enclosed by the curve y = f(x), the x-axis, the line x = a and the line x = b is revolved through 360° about the x-axis is given by

$$V_x = \int_a^b \pi y^2 dx$$



The volume of the solid generated when the region enclosed by the curve x = f(y), the y-axis, the line y = a and the line y = b is revolved through 360 ° about the y-axis is given by

$$V_y = \int_a^b \pi x^2 dy$$

Refer to Note 3:

$$\int x^n dx = \frac{x^{n+1}}{n+1} + c$$

Example:

$$\int x^2 dx = \frac{x^3}{3} + c$$

$$\int ax^n dx = \frac{ax^{n+1}}{n+1} + c$$

Example:

$$\int 2x^3 dx = \frac{2x^4}{4} + c = \frac{x^4}{2} + c$$

Example:

$$\int \frac{1}{x^5} dx = \int x^{-5} dx = \frac{x^{-4}}{-4} + c$$

Example:

$$\int \sqrt{x} dx = \int x^{\frac{1}{2}} dx = \frac{x^{\frac{3}{2}}}{\frac{3}{2}} + c = \frac{2x^{\frac{3}{2}}}{3} + c$$

$$\int (u \pm v) dx = \int u dx \pm \int v dx$$

u and v are functions in x

Example:

1. $$\int 3x^2 + 2x dx = \int 3x^2 dx + \int 2x dx$$
$$= \frac{3x^3}{3} + \frac{2x^2}{2} + c = \frac{\cancel{3}x^3}{\cancel{3}} + \frac{\cancel{2}x^2}{\cancel{2}} + c$$
$$= x^3 + x^2 + c$$

Example:

$$\int (x+2)(3x+1)dx = \int (3x^2+7x+2)dx$$
$$= \int 3x^2dx + \int 7xdx + \int 2dx$$
$$= \frac{3x^3}{3} + \frac{7x^2}{2} + 2x + c$$
$$= x^3 + \frac{7x^2}{2} + 2x + c$$

Example:

$$\int (\frac{3x^3+x^2-x}{x})dx = \int (3x^2+x-1)dx$$
$$= \int 3x^2dx + \int xdx - \int 1dx$$
$$= \frac{3x^3}{3} + \frac{x^2}{2} - x + c$$
$$= x^3 + \frac{x^2}{2} - x + c$$

$$\int (ax+b)^n dx = \frac{(ax+b)^{n+1}}{a(n+1)} + C$$

Example:

$$\int (3x+5)^3 dx = \frac{(3x+5)^4}{3(4)} + c$$
$$= \frac{(3x+5)^4}{12} + c$$